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The quantum acoustomagnetoelectric effect due to Rayleigh sound waves

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Abstract. We have developed a theory of the quantum acoustomagnetoelectric effect due to Rayleigh sound waves in the presence of a quantizing magnetic field normal to the crystal–vacuum interface. Electrons are assumed to be specularly reflected at the sample surface. For this case we predict the appearance of an acoustoelectric current along the 'Hall' direction orthogonal both to the magnetic field and to the wavevector of the Rayleigh sound. We also predict quantum oscillations of this current analogous to the de Haas–van Alphen and Shubnikov–de Haas oscillations. We show that in the quantum strong-field limit the current oscillations cease to be a small correction to the monotonic part of the current and can have a large amplitude.

1. Introduction

The Rayleigh sound waves propagating along the stress-free surface of an elastic medium have attracted much attention during the past two decades because of their utilization in acoustoelectronics. Considerable interest in such waves has also been stimulated by the possibility of their use as a powerful tool for studying the electronic properties of surfaces and thin layers of solids.

It is well known that the propagation of Rayleigh sound waves in conductors is accompanied by the transfer of their energy and momentum to conduction electrons. This leads to the emergence of a longitudinal acoustoelectric effect, i.e. a stationary electric current running in a sample in the direction opposite to that of the wave. At present this effect has been studied in detail both theoretically and experimentally, and has found wide application in radioelectronic systems [1].

However, in the presence of a quantizing magnetic field the bulk acoustic waves propagating in a conductor can induce another effect, the so-called quantum acoustomagnetoelectric (QAME) effect. It has been predicted by Galperin and Kagan [2] and later observed in bismuth by Salaneck *et al* [3]. This QAME effect occurs under the condition of strong spatial dispersion, $ql \gg 1$ (q is the modulus of the acoustic wavevector and l is the electron mean free path), when the quantum regime of sound absorption is realized. In this case the interaction between the electrons and the acoustic wave should be treated in terms of collisions of acoustic quanta with electrons. In a magnetic field $\boldsymbol{H}$ satisfying the conditions

$$\hbar\omega_c \gg T \qquad \omega_c \gg \nu \tag{1}$$

($\omega_c = eH/mc$ is the cyclotron frequency, T is the temperature in energy units and ν is the frequency of the electron collisions), the absorption of acoustic quanta by electrons is accompanied by the displacement of the cyclotron orbit centres in the direction perpendicular to both $\boldsymbol{H}$ and $\boldsymbol{q}$ (the 'Hall' direction). It leads to the appearance of the acoustoelectric current component in the same direction, and this is the essence of the QAME effect.

As has been shown in [2, 3] for spherical electron energy surfaces, the QAME effect can occur only in the Voigt configuration ($\boldsymbol{q} \perp \boldsymbol{H}$) when sound quanta induce inter-Landau-level electron transitions. Meanwhile, such transitions are actually forbidden by the energy and momentum conservation laws. In fact, the inter-Landau-level transitions are connected with a change of energy by the amount $\hbar\omega_c$, which can be compensated neither by the energy of the absorbed phonon nor by the alteration of the electron motion energy along the magnetic field because of their smallness. Therefore, in a collisionless approximation (i.e. in neglecting electron relaxation processes), only intra-Landau-level transitions are allowed. Certainly, if we take into account electron scattering processes, inter-Landau-level transitions become possible too. However, their relative contribution to the sound absorption will be small by the parameter $1/ql$. Thus, in the Voigt configuration the QAME effect due to bulk acoustic waves is absent in materials with a spherical Fermi surface (at least in the collisionless regime).

As will be shown below, the situation is quite different for Rayleigh waves propagating in a magnetic field normal to the crystal surface. As a result of the Rayleigh wave spatial localization in the acoustic 'skin layer' (the thickness of which is of the order of the wavelength), the Rayleigh-phonon wavevector component normal to the surface is uncertain. Therefore, if the electron absorbs the Rayleigh phonon, the electron-momentum projection on the magnetic field is not conserved. The result is that intra-Landau-level electron transitions with a change of the position of the cyclotron orbit centre turn out to be possible even for the collisionless regime. These electron transitions account for the QAME effect.

In this paper we present a theory of the QAME effect due to Rayleigh waves. We restrict our consideration to the case of specular reflection of electrons at a crystal surface. We assume that electron energy surfaces are spherical and consider only the deformation mechanism of electron–Rayleigh-phonon interaction. We also suppose that the mechanism that limits the electron mean free path is scattering on randomly distributed point defects (impurities) in the bulk of the crystal. Within the framework of this model we have analysed the magnetic-field dependence of the 'Hall' acoustoelectric current both for the quasiclassical case, when the splitting of electron levels in the magnetic field is considerably less than the Fermi energy ε_F, and for the quantum limit case, when $\hbar\omega_c$ is in the order of ε_F. We show that the current experiences quantum oscillations when the magnetic field is varied. These oscillations are analogous to the de Haas–van Alphen oscillations of magnetic susceptibility and to the Shubnikov–de Haas oscillations of electric conductivity. In the quasiclassical case the considered oscillations have a sinusoidal form and are characterized by a small amplitude as compared with the monotonic part of the current. In the quantum limit case the oscillations become strong, so that the maximum value of the current can considerably exceed its minimum value†

† It should be noted that quantum oscillations of the Rayleigh wave absorption in a magnetic field normal to the crystal surface were predicted by Grishin and Kaner [4] and observed in Ga by Bellessa [5]. These oscillations, however, have an essentially non-sinusoidal form and belong to another type—the so-called Gurevich–Skobov–Firsov giant quantum oscillations. They can be observed only in materials with an extremely large electron relaxation time and/or at very high frequencies of acoustic waves [6].

2. Electron–Rayleigh-phonon interaction in a quantizing magnetic field

Let us suppose that the Rayleigh wave of frequency ω_q is propagating along the surface of a conductor. For simplicity the latter can be viewed as an isotropic elastic medium. We choose the Cartesian coordinate system x, y, z so that the Oy axis coincides with the wavevector $\boldsymbol{q}$ and the Oz axis is oriented along the magnetic field $\boldsymbol{H}$. We also suppose that the crystal occupies the half-space $z \geqslant 0$, and that inequalities (1) hold, i.e. the quantization of electron motion in a magnetic field is essential.

We consider the most realistic case from the point of view of a low-temperature experiment, when

$$\omega_q/\nu = c_{\mathrm{R}}|q|/\nu \ll 1 \qquad ql = qv_{\mathrm{F}}/\nu \gg 1 \tag{2}$$

where c_{R} is the velocity of the Rayleigh wave and v_{F} is the velocity of the electrons on the Fermi surface. The compatibility of these conditions is provided by the smallness of the sound velocity in comparison with the characteristic velocity of the Fermi electrons.

If conditions (2) are satisfied, a macroscopic approach to the description of the acoustoelectric effect is inapplicable, and the problem should be treated by using quantum-mechanical methods. Therefore, we have at first to find the Hamiltonian $\mathcal{H}_{\mathrm{e-p}}$ describing the interaction of the electrons with the Rayleigh phonons. We assume that the interaction is due exceptionally to the deformation produced by the Rayleigh waves. Strictly speaking, it is valid only if $q\delta \gg 1$ (δ is the thickness of the electromagnetic skin layer at a sound frequency for an anomalous skin effect), when we can disregard the conversion of the sound wave into an electromagnetic one. As shown in [7], in the opposite limiting case, $q\delta \ll 1$, the electromagnetic contribution to the electron–Rayleigh-wave interaction becomes comparable with that of the deformation mechanism and, generally, it cannot be neglected. However, we will ignore it, since the character of the considered effect is determined not so much by the concrete mechanism of the electron–phonon interaction as by the peculiarity of the conduction electron dynamics in a magnetic field. On the other hand, it means that the results obtained below cannot claim quantitative exactness, and should be regarded as an order-of-magnitude estimate of the expected effect only.

Using the well known expression for the displacement vector $\boldsymbol{u}(\boldsymbol{r}, t)$ in the Rayleigh wave [8], we present the Hamiltonian $\mathcal{H}_{\mathrm{e-p}}$, following [9–11], in the form

$$\mathcal{H}_{\mathrm{e-p}} = \Lambda \int \Psi^{+}(\boldsymbol{r})\nabla\boldsymbol{u}(\boldsymbol{r}, t)\Psi(\boldsymbol{r})\,\mathrm{d}^3r = \sum_{\alpha,\beta,q}[C_q U_{\alpha\beta}(q) b_q \exp(-\mathrm{i}\omega_q t) + \mathrm{HC}]a_{\alpha}^{+}a_{\beta} \tag{3}$$

where

$$C_q = \mathrm{i}\Lambda c_l^2(\hbar\omega_q^3/2\rho_0\Xi S)^{1/2} \tag{4}$$

$$\Xi = q\left[\frac{1+\sigma_l^2}{2\sigma_l} + \left(\frac{\sigma_l}{\sigma_t} - 2\right)\frac{1+\sigma_t^2}{2\sigma_t}\right] \tag{5}$$

$$\sigma_l = (1 - c_{\mathrm{R}}^2/c_l^2)^{1/2} \qquad \sigma_t = (1 - c_{\mathrm{R}}^2/c_t^2)^{1/2} \tag{6}$$

Λ is the deformation potential constant, $\Psi^{+}(\boldsymbol{r})$ and $\Psi(\boldsymbol{r})$ are the electron field operators, a_{α}^{+} and a_{α} are the creation and annihilation operators of the electron in the $|\alpha\rangle$ state, b_q is the annihilation phonon operator, $U_{\alpha\beta}(q)$ is the matrix element of the operator $U = \exp(\mathrm{i}qy - \kappa_l z)$, $\kappa_l = (q^2 - \omega_q^2/c_l^2)^{1/2}$ is the spatial attenuation factor of the potential part of the displacement field, c_l and c_t are the velocities of the longitudinal and the transverse bulk acoustic waves, ρ_0 is the mass density of the medium and $S = L_x L_y$ is the surface area.

Expression (3) is valid in the frame of reference moving together with the lattice. Our further evaluations are carried out just in this frame. Note that in obtaining (3) we have neglected the inertial term in the expression for the electron energy in the acoustic field because of its relative smallness (by the parameter c_R/v_F) [12].

We now determine the explicit form of the one-electron state $|\alpha\rangle$ and $|\beta\rangle$ between which there occur transitions induced by the Rayleigh phonons. A rigorous formula for the $|\alpha\rangle$ state can be obtained only in the case of specular reflection of electrons at a crystal surface. As will be shown below, the QAME effect is due to electrons that have a small momentum component p_z in comparison with the Fermi momentum p_F. The reflection of such electrons at the surface can be regarded as specular even in normal metals in which $p_F \sim \hbar/a$ (a is the interatomic distance). It is even more true with respect to electrons in semimetals like Bi and to small electron groups in normal metals, since in this case there is an additional argument in favour of specular reflection, namely, a large de Broglie wavelength of the electrons on the Fermi surface as compared with the quantity a. Therefore, we can choose the boundary condition for the electron wavefunction in the form $\psi_\alpha(\boldsymbol{r}) = 0$ at $z = 0$ according to which the surface $z = 0$ is an infinite potential barrier for the electrons. If the magnetic field is determined by the vector potential $\boldsymbol{A}$ in the Landau gauge $A_x = A_z = 0$, $A_y = Hx$, the solution of the Schrödinger equation satisfying the above boundary condition is written as

$$\psi_\alpha(\boldsymbol{r}) = (2/L_y L_z)^{1/2}\phi_n[(x - X_\alpha)/a_H]\exp(\mathrm{i}p_y y/\hbar)\sin(p_z z/\hbar) \tag{7}$$

where α denotes the set of quantum numbers (n, p_y, p_z), L_z is the normalization length in the z direction, $\phi_n[(x - X_\alpha)/a_H]$ is the oscillator wavefunction centred at $X_\alpha = -a_H^2 p_y/\hbar$, and $a_H = (\hbar/m\omega_c)^{1/2}$ is the magnetic length. The electron energy in the $|\alpha\rangle$ state is

$$E_\alpha = E_n + p_z^2/2m \qquad E_n = (n + \tfrac{1}{2})\hbar\omega_c \tag{8}$$

where p_z can be varied from 0 to $+\infty$.

Using expression (7) it is straightforward to evaluate the matrix elements of the operator U. The result is

$$U_{\alpha\beta}(q) = \delta(p_y', p_y + \hbar q)\chi(p_z - p_z', p_z + p_z')[\delta(n', n)M_{nn}(qa_H/2) + \theta(n' - n)M_{n'n}(qa_H/2) + \theta(n - n')M_{n'n}(qa_H/2)] \tag{9}$$

where

$$\chi(p_z - p_z', p_z + p_z') = (\hbar^2\kappa_l/L_z)\{[(p_z - p_z')^2 + \hbar^2\kappa_l^2]^{-1} - [(p_z + p_z')^2 + \hbar^2\kappa_l^2]^{-1}\} \tag{10}$$

$$M_{n'n}(x) = (n!/n'!)^{1/2}x^{n'-n}\exp(-x^2/2)L_n^{n'-n}(x^2) \tag{11}$$

$\delta(a, b)$ is the Kronecker delta symbol, $\theta(x)$ is the Heaviside step function, and $L_n^{n'-n}(x^2)$ are the associated Laguerre polynomials, which are defined as in [13].

For strong magnetic fields where the inequality $qa_H \ll 1$ is satisfied, the quantity $M_{nn}(qa_H/2)$ in the first term of (9) can be replaced by unity, while the other two terms (non-diagonal in n) can be neglected because of their relative smallness. Formally it follows from the Hilb-type asymptotic formula for $M_{n'n}(qa_H/2)$ provided the argument of function

(11) is smaller than the inverse quantity of the subscript n [13]. But actually, according to the energy conservation law, inter-Landau-level transitions induced by acoustic quanta are completely impossible (see below).

The function $\chi(p_z - p'_z, p_z + p'_z)$ in (9) expresses the non-conservation of the electron-momentum projection on the magnetic field, which can be accounted for by the uncertainty of the Rayleigh-phonon wavevector component normal to the surface. For the bulk acoustic waves with $\boldsymbol{q} \perp \boldsymbol{H}$ the matrix element of the electron transition would contain (instead of the χ function) the Kronecker delta $\delta(p'_z, p_z)$, which expresses the conservation of p_z and makes impossible the sound absorption in the collisionless regime. In contrast, in the case of the Rayleigh waves, as seen from (9), $p_z \neq p'_z$, and the prohibition on the sound absorption is removed.

From (10) it is easy to see that χ, as a function of the transferred momentum, reaches its maximum at $p_z - p'_z = 0$. The range where the χ function essentially differs from zero is of the order of $\hbar\kappa_l$. Thus, the quantity $\hbar\kappa_l$ plays, in a sense, the role of a transverse phonon momentum. Designating the change of the electron-momentum projection on the magnetic field by $\hbar\Delta$ and making use of the energy conservation law

$$(n + \tfrac{1}{2})\hbar\omega_c + (p_z^2/2m) + \hbar\omega_q = (n' + \tfrac{1}{2})\hbar\omega_c + (p_z'^2/2m)$$

we can conclude that in strong magnetic fields when $\kappa_l a_H \ll 1$ (or $q a_H \ll 1$), the collisionless absorption of Rayleigh phonons is possible only if $n = n'$. It is easy to see that in this case the absorption process is due to the electrons with

$$p_z = p_z^{(0)} = (m\omega_q/\Delta) - (\hbar\Delta/2).$$

Because of the uncertainty of the quantity Δ, the uncertainty of $p_z^{(0)}$ (as well as its magnitude) will be of the order of $m\omega_q/\kappa_l$. Thus, in the case of the Rayleigh sound absorption there is no stringent dynamic 'selection rule' in p_z for electrons that interact most effectively with acoustic quanta. However, if conditions (1) and (2) are satisfied, the above-mentioned uncertainty of $p_z^{(0)}$ will still be less than the smearing $\Delta p_z^{(0)}$ of the quantity $p_z^{(0)}$ owing to the electron heat motion ($\Delta p_z^{(0)} \sim mT/p_z^{(0)}$) and to the collisions of electrons with scatterers ($\Delta p_z^{(0)} \sim m\nu/\kappa_l$), and it will also be considerably less than p_F. In consequence, the 'Hall' acoustoelectric current considered below will be created mainly by electrons with a small momentum component normal to the surface as compared with the Fermi momentum: $p_z \ll p_F$.

3. General expression for current density of the QAME effect

The DC electric current associated with the QAME effect arises in the second order of the electron–phonon interaction $\mathcal{H}_{e-p}$. For an accurate estimate of the density of this current it is convenient to start with the Kubo formula for the stationary quadratic response to the AC field [14, 15]:

$$\boldsymbol{j}_{ac} = -\frac{1}{V_0\hbar^2}\left\langle \mathrm{Tr}\int d^3r\, \boldsymbol{j}(\boldsymbol{r}) \int_{-\infty}^{0} dt \int_{-\infty}^{t} dt_1\, [H_{e-p}(t), [H_{e-p}(t_1), \rho^{(0)}]]\right\rangle \quad (12)$$

where V_0 is the normalization volume, $\boldsymbol{j}(\boldsymbol{r})$ is the current density operator, $\rho^{(0)}$ is the statistical operator at the initial moment of time, and

$$H_{e-p}(t) = \exp[(i/\hbar)\mathcal{H}_0 t]\mathcal{H}_{e-p}\exp[-(i/\hbar)\mathcal{H}_0 t] \qquad \mathcal{H}_0 = \mathcal{H}_e + \mathcal{H}_{e-i}.$$

Here the Hamiltonian $\mathcal{H}_e$ describes the electron subsystem and $\mathcal{H}_{e-i}$ is the Hamiltonian of the interaction of electrons with randomly distributed short-range impurities located in the bulk of the crystal. The square and angular brackets in (12) denote the commutator and the averaging over the random position of the scattering centres, respectively, and the symbol Tr indicates the trace of electron and phonon variables.

Using the hypothesis of the adiabatic switching-on of the interaction $\mathcal{H}_{e-p}$ from an infinite past (at $t = -\infty$) and considering the Rayleigh wave as a packet of coherent phonons with the delta-like distribution function $\mathcal{N}(\boldsymbol{k}) = (2\pi)^3 N_q \delta(\boldsymbol{k} - \boldsymbol{q})/V_0$ in the wavevector $\boldsymbol{k}$ space, it is not difficult to get the expression for the acoustoelectric current density component $(j_{ac})_x \equiv j_{ac}$ in the 'Hall' direction. Using (3) and (12) we find

$$j_{ac} = (\hbar\omega_q)^{-1} \int_0^\infty [f(\varepsilon) - f(\varepsilon + \hbar\omega_q)] J(\varepsilon)\, d\varepsilon \tag{13}$$

$$J(\varepsilon) = \frac{2\pi e \omega_q}{V_0} N_q |C_q|^2 \langle \mathrm{Tr_e}[X\delta(\varepsilon + \hbar\omega_q - \mathcal{H}_0)U\delta(\varepsilon - \mathcal{H}_0)U^+ - X\delta(\varepsilon - \mathcal{H}_0)U^+\delta(\varepsilon + \hbar\omega_q - \mathcal{H}_0)U]\rangle \tag{14}$$

where X is the operator of the x component of the cyclotron orbit centre, $f(\varepsilon)$ is the equilibrium Fermi–Dirac distribution function, and $N_q = WV_0/\hbar\omega_q c_R$ is the number of acoustic quanta in the strongly excited Rayleigh phonon mode $|q\rangle$ (W is the density of the acoustic energy flux). We suppose that only one phonon mode has been excited by the external perturbation as is the case in the actual experimental conditions (see, for example [5, 16–18]).

Introducing the resolvents $R^{\pm}(\varepsilon) = 1/(\varepsilon - \mathcal{H}_0 \pm i0)$ of the operator $\mathcal{H}_0$ and expressing the δ functions in (14) by the formula $\delta(\varepsilon - \mathcal{H}_0) = [R^-(\varepsilon) - R^+(\varepsilon)]/2\pi i$, we get after averaging over the random position of the impurities in (14) (as done by Skobov [19])

$$J(\varepsilon) = \frac{2e\omega_q}{\pi V_0} N_q |C_q|^2 \sum_{\alpha,\beta} |U_{\alpha\beta}(q)|^2 X_{\alpha\beta}\, \mathrm{Im}\, G_\alpha(\varepsilon)\, \mathrm{Im}\, G_\beta(\varepsilon + \hbar\omega_q) \tag{15}$$

where $X_{\alpha\beta} = X_\alpha - X_\beta$ and $G_\alpha(\varepsilon)$ is the retarded one-particle Green function averaged over impurity positions

$$G_\alpha(\varepsilon) = \langle R^-_{\alpha\alpha}(\varepsilon)\rangle = [\varepsilon - E_\alpha + i\nu(\varepsilon)/2]^{-1}. \tag{16}$$

Here $\nu(\varepsilon)$ is the probability of the electron scattering per unit time, which satisfies the equation obtained in [19]. However, for simplicity we will further regard the quantity ν as independent of energy and consider it to be a phenomenological constant. Strictly speaking, this approximation holds under the following conditions:

$$\hbar\omega_c \ll (\varepsilon_F T)^{1/2} \qquad \hbar\nu \ll T \tag{17}$$

which restricts the range of validity of the results obtained below by a quasiclassical case when electrons fill a large number of Landau levels ($\varepsilon_F \gg \hbar\omega_c$) and the amplitude of the quantum oscillations of the electron collisions frequency is relatively small. In the quantum case ($\hbar\omega_c \sim \varepsilon_F$) we generally cannot neglect the energy dependence of ν. However we believe that the results obtained in the approximation $\nu = \mathrm{const}$ yield a correct qualitative treatment of the effect under consideration in this case as well. In this connection it is worthwhile to note that the product of the imaginary parts of the one-particle Green functions in (15) arises as the result of the factorization of the two-particle Green function. The factorization holds at all energies except the small vicinity of the point $\varepsilon = E_n$, i.e. except just that narrow range of the ε energy in which the dependence $\nu(\varepsilon)$ is most essential.

The obtained result for the 'Hall' acoustoelectric current, equations (13) and (15), has a form very close to the well known formula for the dissipative current along the electric field in the Shubnikov–de Haas effect. However, there is an essential difference between them, namely, in the latter the quantity $X_{\alpha\beta}$ is squared, while in (15) it is in the first power. It points to a different physical reason for the current rise in these two cases. In the formula of the dissipative current in the crossed electric and magnetic fields $X^2_{\alpha\beta}$ is due to the diffusive migration of the cyclotron orbit centres caused by electron scattering. When there is no scattering the current along the electric field is absent. In contrast, the 'Hall' acoustoelectric current arises because of the lateral displacement (in the $\boldsymbol{H} \times \boldsymbol{q}$ direction) of the cyclotron orbit centres of the electrons upon the continuous absorption of the Rayleigh acoustic quanta and is non-dissipative in origin. In fact, it is easy to see that the 'Hall' current is not equal to zero even in the collisionless limit $\nu \to 0$, when the imaginary parts of the Green function in (15) behave like δ functions of energy. However, as will be shown below, in order to determine the amplitude and shape of the 'Hall' current oscillation peaks arising under magnetic field variations, it is important to take into account the collisions smearing the δ functions.

It should be noted that under 'closed circuit conditions' in the direction of wave propagation apart from the considered 'Hall' current the current component $(j_{ac})_y$ arises, which corresponds to the ordinary longitudinal acoustoelectric effect. Unfortunately, it is difficult to give an accurate estimate for this current component (which is dissipative in origin) using the presented formalism. We think, however, that the ratio $(j_{ac})_x/(j_{ac})_y$ will be of the order of the ratio of conductivities σ_{xy}/σ_{yy}, which is large for the degenerate electron gas by the parameter ω_c/ν.

4. Quantum oscillations of the 'Hall' acoustoelectric current

In the range of comparatively low sound frequencies ($\omega_q \ll \nu$) we can neglect the quantity $\hbar\omega_q$ in the argument of the second G function in (15) and expand the difference of the Fermi functions in (13) by a small shift of their arguments. After substituting (9) and (16) into (15) the summation over n', p'_y and p_y is readily evaluated. Then, we change the summation over p_z and p'_z to integration over the dimensionless variables $\eta = a_H p_z/\hbar$ and $\eta' = a_H p'_z/\hbar$, respectively. As a result, by introducing the dimensionless parameters $\zeta_F = \varepsilon_F/\hbar\omega_c$, $\lambda = \hbar\omega_c/T$, $\Gamma = \nu/2\omega_c$ and the dimensionless variable $\zeta = \varepsilon/\hbar\omega_c$ we find

$$j_{ac} = 2j_0 \left(\frac{\Gamma}{\lambda}\right)^2 \left(\frac{\hbar\omega_q}{T}\right)^3 \sum_n \int_0^\infty d\zeta\, \mathrm{sech}^2[\lambda(\zeta - \zeta_F)/2] \int_0^\infty d\eta\, \eta D(\varepsilon_{n,\eta} - \zeta)$$
$$\times \int_0^\infty d\eta' \eta' D(\varepsilon_{n,\eta'} - \zeta) d(\eta - \eta')[(\eta + \eta')^2 + (\kappa_l a_H)^2]^{-2} \qquad (18)$$

where $\varepsilon_{n,\eta} = n + (1 + \eta^2)/2$ is the electron energy in units of $\hbar\omega_c$,

$$j_0 = We q \kappa_l^2 \Lambda^2 / 4\pi^4 \hbar m \rho_0 \Xi c_R c_l^4$$
$$D(\varepsilon_{n,\eta} - \zeta) = [(\varepsilon_{n,\eta} - \zeta)^2 + \Gamma^2]^{-1}$$
$$d(\eta - \eta') = [(\eta - \eta')^2 + (\kappa_l a_H)^2]^{-2}.$$

Next we point out that under the integral sign over η' in (18) there are two rapidly varying functions of η': $d(\eta-\eta')$ and $D(\varepsilon_{n,\eta'}-\zeta)$. The function $d(\eta-\eta')$ has a maximum at $\eta'=\eta$ with a width of the order of $\kappa_l a_H$. The maximum of the function $D(\varepsilon_{n,\eta'}-\zeta)$ is located at $\varepsilon_{n,\eta'}=\zeta$ and its width with respect to the variable η' is of the order of $\Gamma^{1/2}$. The ratio of these two widths is determined by the parameter $(\hbar\kappa_l^2/m\nu)^{1/2}$. The estimates show that if conditions (1) and (2) are satisfied the parameter is small compared with unity, and, hence, the width of the d function is considerably less than the width of the D function. In contrast, the ratio $d^{\max}/D^{\max}$ determined by the parameter $(m\nu/\hbar\kappa_l^2)^2$ is very large compared with unity. Thus, in the case under consideration the d function is considerably 'sharper' than the D function. This enables us to take the D function outside of the integration sign over η' at the point of the maximum value of the d function. Then, the remaining integral over η' can be evaluated explicitly by means of Cauchy's residue theorem. As a result, the expression for j_{ac} takes the form

$$j_{ac}=(\pi j_0/16)(\Gamma/\lambda)^2(\hbar\omega_q/T)^3(\kappa_l a_H)^{-3}\int_0^\infty d\zeta\,\mathrm{sech}^2[\lambda(\zeta-\zeta_F)/2]\mathcal{F}(\zeta) \qquad (19)$$

where

$$\mathcal{F}(\zeta)=\sum_{n=0}^{\infty}\int_0^\infty \eta^2(\eta^2+\kappa_l^2a_H^2)^{-1}D^2(\varepsilon_{n,\eta}-\zeta)\,d\eta.$$

We will carry out the further analysis of the expression (19) separately for the two limiting cases: the quasiclassical and the quantum one.

4.1. The quasiclassical limit case

In this case the spacing between Landau levels $\hbar\omega_c$ is considerably less than the Fermi energy ε_F and it is convenient to make use of the approach elaborated by Sondheimer and Wilson [20, 21] in the theory of magnetic susceptibility.

It is not difficult to show that $\mathcal{F}(\zeta)$ can be presented as

$$\mathcal{F}(\zeta)=\int_0^\infty \Phi(\varepsilon_{n,\eta})D^2(\varepsilon_{n,\eta}-\zeta)\,d\varepsilon_{n,\eta}$$

where $\Phi(\varepsilon_{n,\eta})$ is the inverse Laplace transformation of the auxiliary function

$$F(\lambda)=\sum_{n=0}^{\infty}\int_0^\infty \eta^2(\eta^2+\kappa_l^2a_H^2)^{-1}\exp(-\lambda\varepsilon_{n,\eta})\,d\eta.$$

In the range of strong magnetic fields where $(\lambda/2)^{1/2}\kappa_l a_H\ll 1$, we have

$$\Phi(\varepsilon_{n,\eta})=(1/4\pi i)(\pi/2)^{1/2}\int_{\gamma-i\infty}^{\gamma+i\infty}\lambda^{-1/2}\sinh^{-1}(\lambda/2)\exp(\lambda\varepsilon_{n,\eta})\,d\lambda. \qquad (20)$$

The singular points of the integrand in (20) are the poles $\lambda_l=\pm i2\pi l$ $(l=1,2,3,\ldots)$ and the branch point $\lambda=0$. By cutting up the complex λ-plane along the negative real axis and deforming the integral contour as indicated in figure 1 we can split $\Phi(\varepsilon_{n,\eta})$ into two parts: a monotonic part determined by the integral on the loop round the cut line, and an oscillating part determined by the sum of residues in the poles. Similarly, the 'Hall'

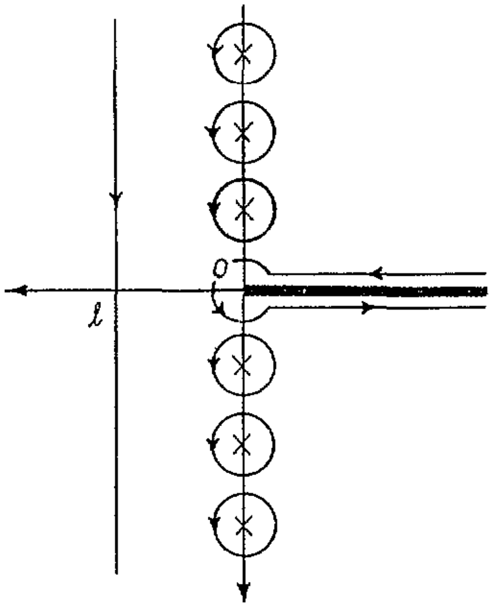

Figure 1. The original contour for the integral appearing in equation (20) is indicated by the vertical line. This contour is deformed into the set of circles around the poles, which are denoted by crosses, and into the loop around the cut line along the negative real axis.

acoustoelectric current is also decomposed into two parts: monotonic (j_{ac}^{mon}) and oscillating (j_{ac}^{osc}). After straightforward calculations we get

$$j_{ac}^{mon} = \pi^2 j_0 (2\varepsilon_F/\hbar\omega_c)^{1/2} (2\omega_c/\nu)(\omega_q/2\omega_c)^3 (\kappa_l a_H)^{-3} \tag{21}$$

$$j_{ac}^{osc} = j_{ac}^{mon} (\hbar\omega_c/2\varepsilon_F)^{1/2} \sum_{l=1}^{\infty} (-1)^l l^{-1/2} (1 + \pi l \nu/\omega_c) \exp(-\pi l \nu/\omega_c) \times (2\pi^2 l/\lambda) \sinh^{-1}(2\pi^2 l/\lambda) \cos(2\pi l \zeta_F - \pi/4). \tag{22}$$

For weak magnetic fields or for comparatively high temperatures ($2\pi^2/\lambda \gg 1$) the oscillations actually vanish because of the exponential decrease of terms of the series in (22).

In the region of intermediate magnetic fields ($2\pi^2/\lambda \sim 1$) the order of magnitude of the amplitude of the oscillations is determined by the first terms of the series in (22) for which $l \sim 1$. In this case we obtain the following estimate:

$$j_{ac}^{osc}/j_{ac}^{mon} \sim (\hbar\omega_c/2\varepsilon_F)^{1/2}. \tag{23}$$

Hence, the amplitude of the oscillating part of the current density is small compared with the monotonic part.

Finally, for strong magnetic fields ($2\pi^2/\lambda \ll 1$) the exponential decrease of terms in the series (22) begins only from $l > l_0 = \min(\lambda/2\pi^2, \omega_c/\pi\nu)$ and the amplitude of the oscillations is determined by the sum of a large number of harmonics in (22) for which $2\pi^2 l/\lambda \sim 1$. The number of such terms is of the order of magnitude of the same l_0.

Therefore, compared with the former estimation (23), there appears an additional factor $l_0^{-1/2} l_0 \sim (\hbar\omega_c/T)^{1/2}$, so that we have

$$j_{ac}^{osc}/j_{ac}^{mon} \sim (\hbar\omega_c/T)^{1/2}(\hbar\omega_c/2\varepsilon_F)^{1/2}. \tag{24}$$

The ratio is small on account of the first of the conditions in (17).

Thus, in the quasiclassical limit case the 'Hall' acoustoelectric current can experience only smooth sinusoidal oscillations of the Shubnikov–de Haas type. However, as we will show further, the situation changes in the quantum limit case when $\hbar\omega_c \sim \varepsilon_F$ and the electrons fill only the lower Landau levels.

4.2. The quantum limit case

In this case it is worthwhile to change the integration order in (19) by using the continuity of the integrand, i.e. first integrate over ζ and then over η. As already mentioned the D function has a maximum at $\varepsilon_{n,\eta} = \zeta$. The characteristic scale of variation of this function is small compared with unity. At the same time, the function $\mathrm{sech}^2[\lambda(\zeta - \zeta_F)/2]$ varies smoothly on the interval of $\Delta\zeta \sim 1$ and, therefore, we can take it outside the integral sign over ζ at the point $\zeta = \varepsilon_{n,\eta}$. Then, the remaining integral over ζ can be calculated easily, and provided the condition $\Gamma \ll 1$ is satisfied we obtain

$$j_{ac} = (\pi^2 j_0/2)(\hbar\omega_c/T)(2\omega_c/\nu)(\omega_q/2\omega_c)^3(\kappa_l a_H)^{-3}\sum_n \int_0^\infty \mathrm{d}\eta\, \eta^2(\eta^2 + \kappa_l^2 a_H^2)^{-1}$$
$$\times (1 + \Gamma/\pi\varepsilon_{n,\eta})\,\mathrm{sech}^2[\lambda(\varepsilon_{n,\eta} - \zeta_F)/2]. \tag{25}$$

If the magnetic field is such that one of the Landau levels gets into the thin heat layer (of the order of T) near the Fermi level, the quantity j_{ac} reaches a maximum value, which can be estimated from (25) as

$$j_{ac}^{max} \simeq C\pi^2 j_0(\hbar\omega_c/T)^{1/2}(2\omega_c/\nu)(\omega_q/2\omega_c)^3(\kappa_l a_H)^{-3}. \tag{26}$$

Here the numerical factor

$$C = (\pi/2)^{1/2}(1 - 2^{3/2})\zeta(-1/2) \simeq 0.5$$

where $\zeta(-1/2) = -0.2079$ is the value of the Riemann zeta function $\zeta(x)$ at the point $x = -1/2$ [22].

If the magnetic field is such that all values of $n + \frac{1}{2}$ are far from ζ_F the current density j_{ac} reaches its minimum value

$$j_{ac}^{min} \simeq \pi^2 j_0(2\omega_c/\nu)(\omega_q/2\omega_c)^3(\kappa_l a_H)^{-3}. \tag{27}$$

Then, according to (26) and (27), we obtain

$$j_{ac}^{max}/j_{ac}^{min} \sim C(\hbar\omega_c/T)^{1/2}. \tag{28}$$

In semimetals in which the effective electron mass m is small compared with the free-electron mass m_0, the ration (28) can be quite large. For instance, for bismuth ($m = 0.01m_0$) at $H = 50$ kOe and $T = 1.2$ K we have $j_{ac}^{max}/j_{ac}^{min} \sim 10$. By the order of magnitude, this estimate is true also for normal metals if the QAME effect is due to the interaction of the Rayleigh wave with electrons belonging to the small 'pockets' of the Fermi surface.

Thus, we see that in the quantum limit case the 'Hall' acoustoelectric current can experience strong oscillations that are periodic in $1/H$ with period equal to that of the de Haas–van Alphen and Shubnikov–de Haas oscillations $\Delta(1/H) = e\hbar/mc\varepsilon_F$. Here we will not consider the shape of the oscillation peaks since in the quantum limit case the obtained formula (25) for j_{ac} is purely an estimate because it ignores the magnetic-field dependence of the quantities ν and ε_F. However, we believe that in the range of small quantum numbers n the shape of the oscillation peaks will be sinusoidal as in the quasiclassical case. In this connection we note that it is just the same form of oscillation as observed in most experiments on sound absorption in a magnetic field when the condition $\omega_q \ll \nu$ is satisfied [6].

5. Conclusions

The main results of this paper can be summarized as follows.

We have shown that in the case of spherical electron energy surfaces the absorption of Rayleigh acoustic quanta by conduction electrons is accompanied by the displacement of the cyclotron orbit centres in the direction perpendicular both to the magnetic field $\boldsymbol{H}$ and to the wavevector $\boldsymbol{q}$ of the Rayleigh sound. It leads to the increase of the QAME effect, i.e. of the DC acoustoelectric current in the 'Hall' direction $\boldsymbol{H} \times \boldsymbol{q}$. The effect occurs already in the collisionless regime of Rayleigh sound absorption and is impossible under analogous conditions for bulk acoustic waves.

Our analysis shows that if the magnetic field strength is varied the 'Hall' acoustoelectric current experiences oscillations with a period characteristic of the de Haas–van Alphen and Shubnikov–de Haas oscillations. In the most realistic case of low sound frequencies ($\omega_q \ll \nu$) the oscillation peaks have a sinusoidal shape and their relative amplitude grows with the increase of the magnetic field strength. In the quantum limit case when the electron level splitting in the magnetic field is of the order of the Fermi energy, the current oscillations cease to be a small correction to the monotonic part of the current and can have a fairly large amplitude of the order of $(\hbar\omega_c/T)^{1/2}$.

In conclusion, we make some numerical estimates of the order of magnitude of the considered effect. Making use of (4)–(6) and (26) and the parameters of Bi ($m = 0.01m_0$, $\Lambda = 5$ eV, $\rho_0 = 10$ g cm^{-3}, $c_l = 2 \times 10^5$ cm s^{-1}, $c_R = 8 \times 10^4$ cm s^{-1} and $\nu = 10^9$ s^{-1}) at $H = 50$ kOe, $T = 1.2$ K, $W = 0.01$ W cm^{-2} and $\omega_q = 10^8$ s^{-1}, we get $j_{ac}^{max} = 0.1$ μA cm^{-2}, which is not difficult to measure by means of standard techniques. Similarly, we obtain $j_{ac}^{max} = 0.6$ μA cm^{-2} for Ga ($m = 0.6m_0$, $\Lambda = 10$ eV, $\rho_0 = 6$ g cm^{-3}, $c_l = 5\times10^5$ cm s^{-1} and $c_R = 2.4 \times 10^5$ cm s^{-1} [5]) at $H = 100$ kOe and the same meanings of the other parameters. However, in this case $j_{ac}^{max}/j_{ac}^{min} \simeq 2$.

The effect considered can be utilized in studying electron–Rayleigh-phonon interaction as well as for determining the parameters of the electron energy spectrum. Besides, since the diffusiveness of electron scattering at the crystal surface will smear the oscillation peaks of the 'Hall' acoustoelectric current, the form of dependence of $j_{ac}(H)$ may allow some qualitative conclusions to be made concerning the character of electron reflection at the surface.

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